St. Joseph's Journal of Humanities and Science

ISSN: 2347 - 5331

http://sjctnc.edu.in/6107-2/

Some Stochastic Models in Bivariate Measurements

R. Saranraj[a*]
M. Iyappan[b]
Mercy Anthony[a]

ABSTRACT

Model is a device such that it is a look alike original phenomena with which one could understand the Nature of the problem without experience it. There is no meaning in having the knowledge of the population by means of experience. Most of the real life and practice oriented problems are handled by proper understanding of the population in which modeling acts as an Anatomy of the phenomenon. An empirical data is always trying to spell out the information within it. Proper formulation of the data from the general environment to Statistical environment is an essential phase of the research.

Keywords: Stochastic models, statistical measurements.

The activity of drug design and development is a combination of a many and multi faceted activities like Understanding the problem, transform in to technical means, converting to the domain problem to modeling, formulation of the problem in to technical languages, verification of the drug effectiveness through generic tests, validation of the performance, etc., and many more of the similar. While administering a drug, the status of a patient has to be evaluated and a through analysis on health conditions is very essential. A suitable formulation of the bio-systems into mathematical formulation and transforming classic mathematical environment into statistical/empirical situations is required.

Most of the treatments in the Indian context deal with the short term health problems like seasonal fevers, epidemics, influenza, etc. This sort of practice not only harmful to the patient at individual level but also a significant threat to the community health in general. Hence there is a need of focus the attention of researchers on evaluation of Negative impacts of this practice. In this model, it is assumed that, the drug administration is purely on choice of the patient not on the competent medical practioners. The users of the drug are not fully aware on the protocols of drug dosage level, drug administration period, etc. Drugs are consumed erratically and stoppage of drug administration is also abrupt and the other assumptions under the similar conditions.

[a*] Department of Statistics, St.Joseph's College of Arts & Science (Autonomous), Cuddalore, Tamil Nadu, India.
[b] Department of Statistics, Mahendra Arts & Science College (Autonomous), Kalippatti, Tamil Nadu, India.

ASSUMPTIONS OF THE MODEL

In this study we develop a stochastic model to measure the effectiveness of drug under the following assumptions: (i) The patients may get ill health such as cold, fever, headache and similar non chronic and short term diseases at a random time. (ii) The patients is not having required knowledge on the drug administration parameters such as dosage level of drug, frequency of drug administration per unit time, time between two spells of drug administration, neither the patient nor the health care taker on his behalf is aware on the side effects of the drug.

STOCHASTIC MODEL

Let 'X' be a random variable of positive impact of drug as the effectiveness of drug is influenced by many factors; The chance of drug effectiveness has to be assessed through uncertainty measures. Hence we can assume 'X' as a Bernoulli variate.

If X = 1; if the drug has positive impact with chance p_1 and X = 0; if the drug has no positive impact with chance p_0. The probability distribution for X is P(X=0) = p_0 and P(X=1) = p_1; where $p_1, p_0 \geq 0$ and $p_1 + p_0 = 1$.

The statistical measures of X can be obtained using the following raw moments

$$\mu_r^1(X) = E(X^r) = \sum_{x=0}^{1} x^r.p(x) = 1^r.p_1 + 0^r.p_2 = p_1$$

$$\Rightarrow \quad \mu_r^1(X) = p_1$$

Let 'Y' be a random variable dealing the event of negative/adverse impact of drug. If Y = 1; if the drug has negative impact with chance q_1 and Y = 0; if the drug has no negative impact with chance q_0. The probability distribution for the variable Y is P(Y = 1) = q_1 and P(Y = 0) = q_0 where $q_1, q_0 \geq 0$ and $q_1 + q_0 = 1$.

The statistical measures of Y can be obtained using the following raw moments,

$$\mu_r^1(Y) = E(Y^r) = \sum_{x=0}^{1} y^r.p(y) = 1^r.q_1 + 0^r.q_2 = q_1$$

$$\Rightarrow \quad \mu_r^1(Y) = q_1$$

We assume X and Y are independent random variables. The joint probability distribution of X, Y is P(X=x, Y=y) = P(X=x). P(Y=y) where x = 0, 1; y = 0, 1. The statistical measures for the Joint probability distribution becomes,

$$\mu_r^1(X,Y) = \sum_{x=0}^{1}\sum_{y=0}^{1}(x.y)^r.p(x,y)$$

Let Z be another random variable which can measure the overall drug effectiveness by considering both positive and negative impacts together. Z is defined with a linear relation of X, Y and is defined as Z = a X + b Y; where 'a' is the coefficient associated with positive impact, Most of the time it is a quantitative measure of drugs positive influence on the health improvement and 'b' is the coefficient associated with Negative impact; usually it is a quantitative measure of adverse effects on the health in the process of treatment we have to consider b as negative value. Therefore the value of Z becomes Z = a X– b Y. In order to obtain various statistical measures of the overall impact of drug, the study has to be focused on Z variable only.

STATISTICAL MEASURES

The raw moments of positive effectiveness,

$$E(X) = \sum_{x=0}^{1} x.p(x) = 0.p_0 + 1.p_1 = p_1 = \mu_1'(x)$$

The raw moments of negative effectiveness,

Y	0	1
P(Y)	q_0	q_1

$$E(Y^r) = \sum_{y=0}^{1} y^r p(y) = 0^r.q_0 + 1^r.q_1 = q_1 = \mu_r'(y)$$

Raw moments of overall drug effectiveness,

Let $Z = aX - bY$

$$\mu_r'(Z) = E(Z^r) = E(aX - bY)^r$$

$$= \sum_{k=0}^{r}(-1)^k (r_k) b^k a^{r-k} E(Y^k) E(X^{r-k})$$

$E(X^{(r-K)}) = p_1$; When $r - k \geq 1$

$=1$; *When* $r = k$

$E(Y^k) = q_1$; when $k \geq 1$

$=1$; *When k = 0*

The first order raw moment of Z is,

$$\mu_r'(Z) = E(Z^r) = \sum_{k=0}^{r}(-1)^k \binom{r}{k} b^k a^{r-k} E(Y^k) E(X^{r-k})$$

$$E(z) = a.E(X) - b.E(Y)$$

$$\mu_1'(Z) = ap_1 - bq_1$$

The second order raw moment of Z is,

$$E(Z^2) = \mu_2'(Z) = \sum_{k=0}^{2}(-1)^k \binom{2}{k} b^k a^{2-k} E(Y^k) E(X^{2-k})$$

$$\mu_2'(Z) = a^2 p_1 - 2abp_1q_1 + b^2 q_1$$

The third order raw moment of Z is,

$$E(Z^3) = \mu_3'(Z) = \sum_{k=0}^{3}(-1)^k \binom{3}{k} b^k a^{3-k} E(Y^k) E(X^{3-k})$$

$$\mu_3'(Z) = (a^3 p_1 - b^3 q_1) - 3abp_1q_1(a-b)$$

The fourth order raw moment is,

$$E(Z^4) = \mu_4'(Z) = \sum_{k=0}^{4}(-1)^k \binom{4}{k} b^k a^{4-k} E(Y^k) E(X^{4-k})$$

$$\mu_4'(Z) = E(Z^4) = a^4 p_1 + b^4 q_1 + 2abp_1q_1(3ab - 2a^2 - 2b^2)$$

The second order central moment of Z is,

$$\mu_2(Z) = \left[a^2 p_1 - 2abp_1q_1 + b^2 q_1\right] - [ap_1 - bq_1]^2$$

$$\mu_2(Z) = a^2 p_1 p_0 + b^2 q_1 q_0$$

The third order central moment of Z is,

$$\mu_3(Z) = \left[(a^3 p_1 - b^3 q_1) - 3abp_1q_1(a-b)\right]$$
$$-3\left[a^2 p_1 - 2abp_1q_1 + b^2 q_1\right]$$
$$[ap_1 - bq_1] + 2[ap_1 - bq_1]^3$$

$$\mu_3(Z) = a^3 p_1 (1 - 3p_1 + 2p_1^2) - b^3 q_1 (1 - 3q_1 + 2q_1^2)$$

The fourth order central moment of Z is,

$$\mu_4(Z) = a^4 p_1 \left[1 - 4p_1 + 6p_1^2 - 3p_1^3\right] +$$
$$b^4 q_1 \left[1 - 4q_1 + 6q_1^2 - 3q_1^3\right] +$$
$$6a^2 b^2 p_1 q_1 [p_1 - q_1 + p_1 q_1]$$

Mean $= \mu_1'(Z) = ap_1 - bq_1$

Variance $= \mu_2(Z) = a^2 p_1 p_0 + b^2 q_1 q_0$

Standard Deviation (S.D) is,

$$[a^2 p_1 p_0 + b^2 q_1 q_0]^{\frac{1}{2}}$$

Coefficient of Variation (C.V) is,

$$\frac{[a^2 p_1 p_0 + b^2 q_1 q_0]^{\frac{1}{2}}}{ap_1 - bq_1}$$

Coefficient of Skewness (β1) is,

$$\frac{[a^3 p_1 (1 - 3p_1 + 2p_1^2) - b^3 q_1 (1 - 3q_1 + 2q_1^2)]^2}{[a^2 p_1 p_0 + b^2 q_1 q_0]^3}$$

Coefficient of Kurtosis (β2) is,

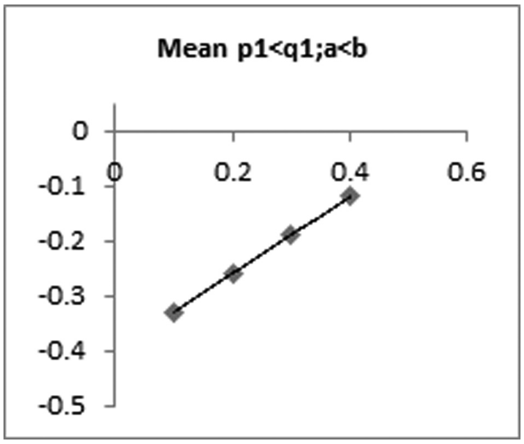
$$\frac{[a^3 p_1 (1 - 3p_1 + 2p_1^2) - b^3 q_1 (1 - 3q_1 + 2q_1^2)]^2}{[a^2 p_1 p_0 + b^2 q_1 q_0]^{\frac{3}{2}}}$$

NUMERICAL ILLUSTRATIONS

The numerical data is placed in tables and also for more understanding purpose, graphs were constructed to mean and variances and they were presented.

Table 1:Values of Mean, Variance, Coefficient of Skewness and Kurtosis for varying values of p_1, q_1, a and b

p_1	q_1	a	b	Mean	Variance	C.V	BETA1	BETA2
0.1	0.5	0.7	0.8	-0.33	0.204	-1.369	0.072	2.01
0.2	0.5	0.7	0.8	-0.26	0.238	-1.878	0.08	2.126
0.3	0.5	0.7	0.8	-0.19	0.263	-2.699	0.046	2.07
0.4	0.5	0.7	0.8	-0.12	0.278	-4.391	0.013	2.007



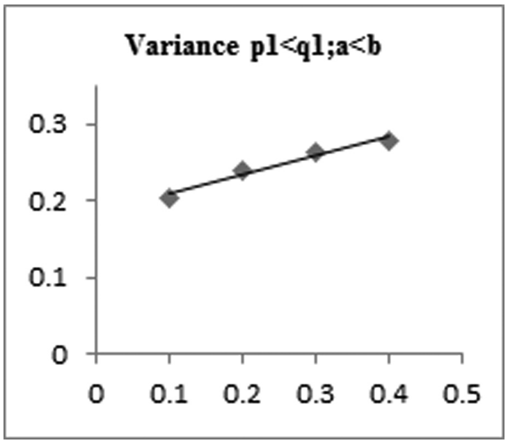
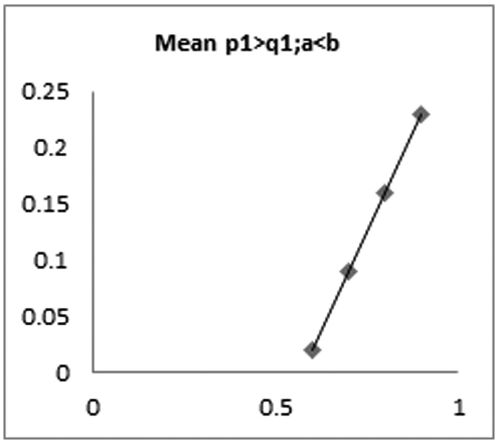
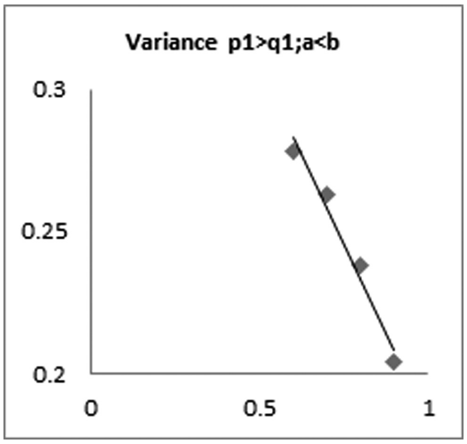


Table 2: Values of Mean, Variance, Coefficient of Skewness and Kurtosis for varying values of p_1, q_1, a and b

p_1	q_1	a	b	Mean	Variance	C.V	BETA1	BETA2
0.6	0.5	0.7	0.8	0.02	0.278	26.344	0.013	2.007
0.7	0.5	0.7	0.8	0.09	0.263	5.697	0.046	2.07
0.8	0.5	0.7	0.8	0.16	0.238	3.052	0.08	2.126
0.9	0.5	0.7	0.8	0.23	0.204	1.964	0.072	2.01





CONCLUSIONS

It is observed that the average drug efficiency is an decreasing function of p_1, and it is negative when $p_1 < q_1$, $a < b$. The variability of drug efficiency is an increasing function of p1, and it is positive when $p_1 < q_1$, $a < b$. The coefficient of variation is a decreasing function of p_1, and it is negative when $p_1 < q_1$, $a < b$. By varying different values of p_1, we obtain corresponding values of beta 1 and beta 2 values (Table:1).

By taking various values of p_1 and keeping the remaining values as constant, it is observed that the average drug efficiency is an increasing function of p_1, and it is positive when $p_1 > q_1$, $a < b$ .The variability of drug efficiency is an decreasing function of p_1, and it is positive when $p_1 > q_1$, $a < b$. The coefficient of variation is a decreasing function of p_1, and it is positive when $p_1 > q_1$, $a < b$. By varying different values of p_1, we obtain corresponding values of beta 1 and beta 2 values (Table:2).

REFERENCES

- Andrez Swierniak (2004); Infinite Dimensional Models applicable in modeling and analysis of cancer chemotherapy; Journal of Medical Informatics and technologies; Vol:8; PP:1-8.
- W.E.Evans, H.L.McLeod (2003) Pharmacogenomics-drug disposition, drug targets and side effects. New England J Med.Vol.348 ☺2003); PP: 538-549.
- Khalili .S, Monaco.J.M, Armaou.A (2010); Development of a stochastic model for the efficacy of NRTIs using known mechanisms of action; J.Theor.Biol;Vol; Vol 265(4);PP:704-717.
- P.Tirupathi Rao, K.Srinivasa Rao, Padmalatha. K (2010); Stochastic Models for optimal drug administration in cancer chemotherapy; International Journal of Engineering Science and Technology; Vol.2 (5); PP: 859-865.
- P.Tirupathi Rao, R. Bharathi (2010) ;Stochastic modeling in the evaluation of drug efficiency, International Journal of Statistics and System; Vol .5,No.3(2010); PP:427-438.
- Wong D, Modi R, Ramanathan M (2003); Assessment of Markov –dependent stochastic models for drug administration compliance; Clin Pharmacokinet; Vol 42(2); PP: 193-204.
- Xiaohong Qi (2005); Stochastic Models for Prodrug Targeting 1.Diffusion of the Efflux Drug; Molecular Pharmaceutics; Vol 3; No 2; PP: 187-195.